

Do Maxicharged particles exist?

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Abstract

The critical charge Z_c is estimated for elementary particles using a Newton-Wigner position operator inspired model. Particles with $Z \sim Z_c$ (maxicharged particles), if they exist at all, can have unusual properties which turn them into illusive objects not easy to detect. Dirac's magnetic poles have a (magnetic) charge $g \gg Z_c$. This gives one more argument that it is unexpected that pointlike monopoles to be found in our world, where $\alpha^{-1} \simeq 137$.

The aim of this brief note is to raise a question, rather than to give the answer on it. Why all observed elementary (not composite) particles have small electric charge $|Z| \leq 1$? May elementary particles with $|Z| > 1$ exist?

This question can be considered as a one more aspect of the known charge quantization mystery. Although this quantization can be understood in the framework of grand unification theories [1] or even in the Standard model [2], the most elegant explanation dates back to Dirac's seminal paper [3] on magnetic monopols. Neither of these approaches actually exclude the existence of multicharged particles.

As small electric charges can more easily escape detection than big charges, theorists are more willing in introducing the former in their theories. So in the literature such exotics can be found as millicharged [4] or minicharged [5] particles. They were experimentally searched [6], but not yet found. As for multicharged particles, only a few (to our knowledge) examples were suggested. Doubly charged Higgs boson was introduced in [7] and doubly charged (but composite) lepton in [8]. Neither of them were found at yet [9].

At least one reason can be imagined which makes big charges uncomfortable. It is well known [10, 11] that, when a nucleus charge increases, ground state electron energy level in its Coulomb field lowers and for some critical value of the charge, $Z_c \simeq 170$, plunges into the Dirac's sea of negative energy levels. After this the vacuum becomes unstable. So Z_c determines an "electrodynamical upper frontier" for the periodic system of chemical elements.

But a finite size of the nucleus, which removes Coulomb field singularity at the origin, plays an important role in reaching such a conclusion and in calculation of Z_c : The Dirac's equation with bare Coulomb potential becomes illdefined for $Z > 137$. And fundamental elementary particles (quarks, leptons,...) are believed to be pointlike. So at first sight the above described notion of critical charge does not make sense for them.

However an arbitrarily precise localization is impossible for a relativistic particle, as was realized a long time ago [12]. This means that in relativistic theory an elementary particle no longer can be considered as a pointlike source for the Coulomb field.

The meaning of the localization for relativistic particles was carefully investigated [13, 14]. In particular, the most localized wave-packet for spin zero particle with mass m , which does not contain any admixture of negative frequencies, is given by the Newton-Wigner wave function [13]

$$\psi(r) \sim \left(\frac{m}{r}\right)^{5/4} K_{5/4}(mr) , \quad (1)$$

where $K_\nu(r)$ is a modified Bessel function.

Unfortunately, $\psi(r)$ in (1), belonging to the continuous spectrum, is not normalizable and diverges at the origin as $r^{-5/2}$. But it can not be expected that the one particle picture, which is assumed in (1), remains valid for distances $r \ll m^{-1}$. Therefore, we may consider the following simple model for pointlike elementary particle with electric charge Ze ,

$$(Ze)^{-1}\rho(r) = \begin{cases} 0 & , \text{ if } r \leq r_0 \\ Cr^{-5/2}K_{5/4}^2(mr) & , \text{ if } r > r_0 \end{cases} . \quad (2)$$

Here $\rho(r)$ stands for charge density at a point \vec{r} , and the constant C is determined from the normalization condition

$$4\pi \int_0^\infty \rho(r)r^2 dr = Ze \quad (3)$$

The cutoff parameter r_0 must obey $r_0 \ll m^{-1}$. We have somewhat arbitrarily take $r_0 = 0.01m^{-1}$. The prescription $\rho = 0$ when $r \leq r_0$ is a reflection of our desire Eq.2 to resemble topological soliton model for electron [15]. Instead we may take $\rho(r) = \text{const} \equiv \rho(r_0)$ for $r \leq r_0$. the results do not change significantly for massive enough particles and for the lightest particle, still in the realm of our interest, the difference does not exceed 15%. Having in mind a qualitative nature of our argumentation, such subtleties will be left beyond our care. Note that in [15] r_0 coincides with electron classical radius $(137m)^{-1}$, so giving some justification for our choice. If some charge e_1 probes the spherically symmetric charge distribution (2), the potential energy of their interaction is

$$V = -4\pi\alpha \left[\frac{1}{r} \int_0^r x^2 \rho(x) dx + \int_r^\infty x \rho(x) dx \right] , \quad (4)$$

where $\alpha = \frac{Z|ee_1|}{4\pi}$, and opposite sign charges were assumed.

Now we are inclined to consider Dirac's equation, with the potential defined from (2÷4), for the ground state energy level in the situation when this level just dived into the negative energy sea, that is $E=-1$, in units for which the probe particle mass $m_1 = 1$. For $m \gg m_1$, this equation for the radial function G looks like [10]

$$\ddot{G} - \frac{\dot{V}}{V} \dot{G} + \left[V(V+2) + \frac{1}{r} \frac{\dot{V}}{V} \right] G = 0 , \quad (5)$$

where points designate derivatives, for example, $\dot{G} = \frac{dG}{dr}$.

By substitution $G(r) = \sqrt{V(r)}\psi(r)$, this equation takes the form which is more convenient for numerical calculations

$$\ddot{\psi} + \left[V(V+2) + \frac{1}{r} \frac{\dot{V}}{V} + \frac{\ddot{V}}{2V} - \frac{3}{4} \left(\frac{\dot{V}}{V} \right)^2 \right] \psi = 0 . \quad (6)$$

For large distances $r \gg m^{-1}$, $K_{5/4}^2(mr)$ in (2) falls as e^{-2mr} . Therefore the second term in (4) can be dropped for such distances and the first term, because of the normalization condition (3), gives just the Coulomb potential $V(r) = -\alpha/r$, for which equation (5) is exactly solvable in terms of the modified Bessel function of complex index [10]

$$G(r) \sim K_{i\nu}(\sqrt{8\alpha r}) , \quad \nu = 2\sqrt{\alpha^2 - 1} . \quad (7)$$

Let us take some $R \gg (2m)^{-1}$. Equation (5) (in fact(6)) can be numerically solved in the region $0 \leq r \leq R$ subject to the boundary conditions $G(0)=0$, $\dot{G}(0) \neq 0$. Then the smoothness of the logarithmic derivative at $r = R$ gives an equation which determines the critical coupling α_c :

$$\left. \frac{z \frac{dK_{i\nu}(z)}{dz}}{K_{i\nu}(z)} \right|_{z=\sqrt{8\alpha}r} = \left. \frac{2R\dot{G}(r)}{G(r)} \right|_{r=R} . \quad (8)$$

The critical coupling so evaluated shows weak dependence on the mass m and changes from $\alpha_c \simeq 1.03$ for $m = 10^4$ to $\alpha_c \simeq 1.1$ for $m = 20$. These numbers correspond to the choice $R = 10m^{-1}$. If we take $R = 5m^{-1}$ instead, the modifications don't exceed a few percent. Roughly modeling particle-antiparticle situation by setting $m=2$, we find $\alpha_c \approx 2.5$.

We infer the following main conclusion from the above considerations: every pointlike electric charge Ze , such that $\frac{Z^2 e^2}{4\pi} \approx \frac{Z^2}{137} > 2 \div 3$ destabilizes the vacuum.

The actual value of α_c can be even smaller, if we remember that field theoretical effects decrease Z_c in the case of nucleus [16] and some investigations show that chiral phase transition is expected in strongly coupled QED for $\alpha_c \approx \frac{\pi}{3}$ [17].

In any case in the following we will treat $\alpha_c \approx 2 \div 3$ as a fair estimate. So $Z_c \approx 15 \div 20$ can be considered as an "electrodynamical upper frontier" for pointlike elementary particles.

But there is quite a lot space from 1 to Z_c . Where are particles inhabiting this interval?

Particles with $\alpha \approx \frac{Z^2}{137} > 1$ (we will call them maxicharged particles) are of particular interest, because their interactions are essentially nonperturbative. For example, an "onium" from such a particle and antiparticle will decay more willingly into $(n+1)$ photons than into n photons, because now $Ze > 1$. This means that in fact it decays into an infinite number of soft photons, that is into a classical field.

Another remarkable property of the maxicharged particles is that their classical radius $r_0 = \frac{\alpha}{m}$ ($\alpha \approx Z^2/137$) is bigger than their quantum size (Compton wavelength) $\lambda = \frac{1}{m}$. Because of this property it is not very easy to produce them in, for example, electron positron collisions. If $\tau \sim \frac{1}{m}$ is the production time of maxicharged particle-

antiparticle pair and τ_0 their annihilation time, then [18]

$$\frac{\tau}{\tau_0} \sim \alpha \left(\frac{\lambda}{r_0} \right)^3 = \alpha^{-2} < 1 .$$

So the pair is annihilated before they are created [18]! This suggests that maxicharged particles can be rather illusive objects, irrespective of their masses.

In fact, the notion of maxicharged particles was introduced by Schwinger [19]. Below we repeat his arguments from which more clearly defined and shaped out notion of maxicharged particles can be deduced.

Electrodynamics with electric charges e and magnetic charges g reveals the duality symmetry, which can be viewed as a rotation in the (e, g) space. However, this symmetry should be spontaneously violated [20], that is we should have the definite direction for the electric axis in the (e, g) space. In fact this direction can be guessed from the fact that the only small charges surround us in our world [19]. First of all, let us introduce an invariant definition of small charges [19]: we will say that a particle with electric charge e_a and magnetic charge g_a belongs to the category of small charges if

$$\frac{e_a^2 + g_a^2}{4\pi} < 1 . \quad (9)$$

Correspondingly big charges (maxicharged particles in our terminology) are defined through

$$\frac{e_a^2 + g_a^2}{4\pi} \geq 1 . \quad (10)$$

If a and b are an arbitrary pair of small charges, then

$$\left(\frac{e_a g_b - e_b g_a}{4\pi} \right)^2 \leq \frac{e_a^2 + g_a^2}{4\pi} \frac{e_b^2 + g_b^2}{4\pi} < 1 . \quad (11)$$

On the other side, Schwinger's symmetrical quantization condition reads:

$$\frac{e_a g_b - e_b g_a}{4\pi} = n , \quad (12)$$

where n is an integer.

Now (11) and (12) are compatible only if $n = 0$! Therefore, for any pair of small charges we have [19]

$$\frac{g_a}{e_a} = \frac{g_b}{e_b}.$$

This means that small charges occupy a single line in the (e,g) space, and it seems from our every day experience that just this line is chosen as representing electric charge axis after spontaneous breakdown of the duality symmetry. In other words none of small charges possess any amount of magnetic charge. Dyons can live only in the wonderland of maxicharged particles!

Now we turn to more speculative line of reasoning. the most natural symmetrical solution of Dirac's (nonsymmetrical) quantization condition

$$\frac{eg}{4\pi} = \frac{n}{2}, \quad n - \text{integer},$$

would be $e = g$. So in such a hypothetical world singly charged particles will have $\alpha = \frac{e^2}{4\pi} = 0.5$, and doubly charged particles - $\alpha = 2$. Clearly triply charged particles lay beyond the vacuum stability border, if we adopt the above cited value for the critical coupling $\alpha_c \simeq 2 \div 3$. In fact even doubly charged particles look suspicious enough. So maybe the observed absence of multicharged particles is mere reminiscence of the epoch when there was a full harmony between electrical and magnetic forces?

Note that the above picture to have any chance to be valid, something must happen to the scale in the duality space, not only to the orientation of the electric axis, because we know quite well that $\alpha \simeq (137)^{-1}$ and not 0.5! Can we hope that the present value of the fine structure constant is associated to the symmetry breaking between electric and magnetic forces and so can be understood from purely symmetry considerations? Here we have a tempting association that just from conformal (or scale) symmetry considerations Armand Wyler obtained his marvelous formula [21]:

$$\alpha = \frac{9}{16\pi^3} \left(\frac{\pi}{5!} \right)^{1/4} \simeq \frac{1}{137.03608}.$$

(For discussions of this formula, see [22]. Some different "derivations" of this or similar formula can be found in [23], and for other attempts to calculate the fine structure constant, see [24]).

Maybe this "number in search of a theory" [25] at last finds it in electro-magnetic duality and its breaking?

We feel that it is time to finish. Russian folklore says that "one simpleton can ask so much questions that hundred sages fail to answer". The only consolation for us is the hope that questions raised in this essay do not fall into such a category.

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References

- [1] J.C. Pati and A. Salam , Phys. Rev. **D10**, 275 (1974).
H. Georgi and S. Glashow , Phys. Rev. Lett. **32**, 438 (1974).
- [2] K.S. Babu and R.N. Mohapatra , Phys. Rev. Lett. **63**, 938 (1989)
and Phys. Rev. **D41**, 271 (1990).
R. Foot, H. Lew and R.R. Volkas , J. Phys. **G19**, 361 (1993);
Erratum, *ibid.* 1067.
- [3] P.A.M. Dirac , Proc. R. Soc. (London) **A133**, 60 (1931).
For inclusion of fractionally charged particles, see for example
P.A. Horváthy and J.H. Rawnsley , Commun. Math. Phys. **99**,
517 (1985).
- [4] B. Holdom , Phys. Lett. **166B**, 196 (1986).
H. Goldberg and L.J. Hall , Phys. Lett. **174B**, 151 (1986).
S. Davidson and M. Peskin , Phys. Rev. **D49**, 2114 (1994).
M.I. Dobroliubov and A.Yu. Ignatiev , Phys. Rev. Lett. **65**, 679
(1990) and Mod. Phys. Lett. **A8**, 917 (1993).
- [5] A.Yu. Ignatiev , V.A. Kuzmin and M.E. Shaposhnicov , Phys.
Lett. **B84**, 315 (1979).
M.Suzuki , Phys. Rev. **D38**, 1544 (1988).
R.N. Mohapatra and I.Z. Rothstein , Phys. Lett. **B247**, 593
(1990).
K.S. Babu and R.R. Volkas , Phys. Rev. **D46**, R2764 (1992).
M. Maruno , E. Takasugi and M. Tanaka , Prog. Theor. Phys.
86, 907 (1991).
R.N. Mohapatra and S. Nusinov , Int. J. Mod. Phys. **A7**, 3817
(1992).
- [6] E. Golowich and R.W. Robinett , Phys. Rev. **D35**, 391 (1987).
S. Davidson and B.Campbell , Phys. Rev. **D43**, 2314 (1991).
J.A. Jaros , A search for millicharged particles at SLAC , preprint
SLAC-PUB-95-6810.
- [7] R.N. Mohapatra and G. Senjanovic , Phys. Rev. Lett. **44**, 912
(1980).
G.B. Gelmini and M. Roncadelli , Phys. Lett. **B99**, 411 (1981).

- [8] N. Cabibbo , L. Maiani and Y. Srivastava , Phys. Lett. **B139**, 459 (1984).
G. Pancheri and Y.N. Srivastava , Phys. Lett. **B146**, 87 (1984).
F. Nakamura , Phys. Lett. **B167**, 209 (1986).
- [9] A. Accomando and S. Petrarca , Phys. Lett. **B323**, 212 (1994).
OPAL coll.: P.D. Acton , G. Alexander , J. Allison et al., Phys. Lett. **B295**, 347 (1992)
and preprint CERN-PPE/95-021 , 1995.
- [10] V.S. Popov , Yad.Fiz. **12**, 429 (1970).
Ya.B. Zel'dovich and V.S. Popov , Sov. Phys. Usp. **14**, 673 (1972).
- [11] S.S. Gershtein , Ya.B. Zel'dovich , Lett. Nuovo Cimento **1**, 83 (1969).
W. Pieper , W. Greiner , Z.Phys. **218**, 327 (1969).
J. Rafelski , L.P. Fulcher and A. Klein , Phys. Rep. **38C**, 227 (1978).
W. Greiner , B. Müller and J. Rafelski , Quantum Electrodynamics of Strong Fields , Springer-Verlag , Berlin , 1985.
- [12] L. Landau and R. Peierls , Z.Phys. **69**, 56 (1931).
- [13] T.D. Newton and E.P. Wigner , Rev. Mod. Phys. **21**, 400 (1949).
- [14] A.S. Wightman , Rev. Mod. Phys. **34**, 845 (1962).
H. Bacry , Localizability and Space , Lecture Notes in Physics , **vol. 308** , Springer , Berlin , 1988.
- [15] R. Righi and G. Venturi , Lett. Nuovo Cimento **31**, 487 (1981),
and Int. J. of Theor. Phys. **21**, 63 (1982).
For another approach see A. Kovner and B. Rosenstein , Phys. Rev. **D49**, 5571 (1994) (hep-th/9210154).
- [16] Y. Hirata and H. Minakata , Phys. Rev. **D34**, 2493 (1986).
- [17] P.I. Fomin , V.P. Gusynin , V.A. Miransky and Yu.A. Sitenko , Riv. Nuovo Cimento **6**, 1 (1983).
V.A. Miransky , Nuovo Cimento **90A**, 149 (1985).
For recent discussion see for example K. Langfeld , Chiral symmetry breaking in strongly coupled QED?, preprint SACLAY-SPHT-95-096 (hep-ph/9508307).

- [18] T. Datta , Lett. Nuovo Cim. **37**, 51 (1983).
- [19] J. Schwinger , Phys. Rev. **D12**, 3105 (1975).
- [20] F. Strocchi , Phys. Lett. **B65**, 447 (1976).
- [21] A.Wyler , C.R.Acad. Sci. , **A269**, 743 (1969) and **A272**, 186 (1971).
- [22] B. Robertson , Phys. Rev. Lett. **27**, 1545 (1971).
H.M. Schwartz , Lett. Nuovo Cimento **2**, 1259 (1971).
R. Gilmore , Phys. Rev. Lett. **28**, 462 (1972).
R.L. Pease , Int. J. Theor. Phys. **16**, 405 (1977).
See also Physics Today **24**, N8, p.17. (1971).
- [23] G. Rosen , Lett. Nuovo Cimento **37**, 48 (1983).
J.P. Vigier , Lett. Nuovo Cimento **7**, 501 (1973).
G.B. Cvijanovich and J.P. Vigier , Found. Phys.**7**, 77 (1977).
- [24] A.K. Pandey and A.N. Nantri , Lett. Nuovo Cimento **16**, 540 (1976).
G. Rosen , Phys. Rev. **D13**, 830 (1976).
G.F. Chew , Phys. Rev. Lett. **47**, 764 (1981).
D.K. Ross , Int. J. Theor. Phys. **25**, 1 (1986).
L. Motz , Nuovo Cimento **37A**, 13 (1977).
- [25] S.L. Adler , Theories of the fine structure constant α ,
Summary talk delivered at the Third International Conference
on Atomic Physics (Boulder, Colorado, 1972). Batavia preprint
NAL-THY-59.